MATHEMATICS_JULY 2019

1. The characteristic impedance, Z, of a transmission line may be determined from the complex formula $Z^2 = \dfrac{R + j\omega L}{G + j\omega C}$

Determine in the polar form, r∠θ, EACH of the following for a transmission line where R = 1.8 ohms, $\omega = 10^4$, $L = 0.15 \times 10^{-3}$ henrys. $G = 4.4 \times 10^{-6}$ ohms and $C = 1.3 \times 10^{-9}$ farads:

(a) Z^2 (9)

(b) Z, given that from De Moivre's Theorem, $(r\angle\theta)^n = r^n\angle n\theta$, for any value of n; (4)

(c) $\dfrac{1000}{Z}$ (3)

2. (a) Solve the following system of equations which model the currents flowing in an electrical network:

$$1.5i_1 + i_2 - 1.5i_3 = 2$$
$$1.4i_1 - 0.7i_2 + 4.2i_3 = -0.7$$
$$1.3i_1 + 3.9i_2 + 2.6i_3 = 20.8$$

(10)

(b) Express the following as a single algebraic fraction in its simplest form:

$$\frac{a^2 - b^2}{2a - b} \times \frac{2a^2 + ab - b^2}{a^2 + 2ab + b^2}$$

(6)

3. (a) Solve the following equation for $x > 0$, correct to 3 decimal places:

$$\frac{2}{x+1} + \frac{3x}{x-2} = 1$$ (8)

(b) Transpose the following formula to make x the subject:

$$v = \frac{1}{K}\left(\frac{1+\left(\frac{1}{x}+1\right)^2}{Lg}\right)^{\frac{1}{2}}$$ (8)

4. (a) The heat generated by a current in a wire varies directly with the time t, directly with the square of the voltage v and inversely with the resistance R.

When the voltage is 60 volts and the resistance is 45 ohms, the heat generated is 96 units per second.

Calculate the heat generated, in a similar wire, in 1 minute when the voltage is 50 volts and the resistance is 30 ohms. (8)

(b) The bending moment, M, in Newton metres, at a point in a beam is given by

$M = \frac{2.5x(15-x)}{2}$ where x metres is the distance from the point of support.

Calculate the values of x when the bending moment is 62.5 Nm. (8)

5. (a) Draw the graph of the function, $y = \tan x$, in the range $x = 4.40$ to $x = 4.60$ radians, in intervals of 0.05 radians. (10)

Suggested scales: *horizontal axis* 2 cm = 0.05
vertical axis 2 cm = 0.5

(b) Use the graph drawn in Q5(a) to determine the solution of the equation $\tan x = 3.5$. (2)

(c) By drawing a suitable straight line graph on the graph drawn in Q5(a), solve the equation $x = \tan x$. (4)

6. (a) A casting is slung from a horizontal beam by two chains 2.5 metres apart.

The lengths of the chains are 2 metres and 2.3 metres, and both are hooked to the same lifting eye of the casting.

Calculate the angles made by the chains with the beam. (8)

(b) Fig Q6(b) shows a rectangle EFGH, 56 cm × 33 cm, enclosed within rectangle ABCD such that angle HGC = 25°.

Calculate the length of AB. (8)

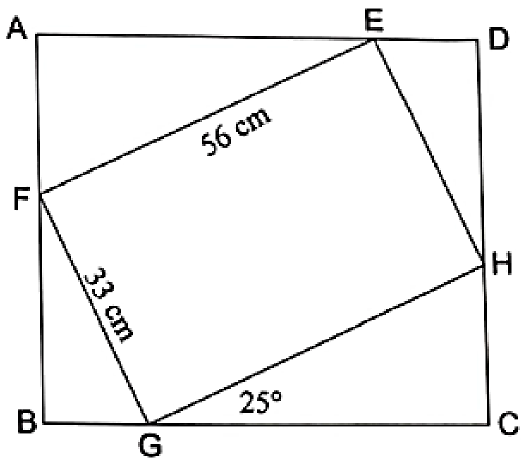

Fig Q6(b)

7. (a) The displacement, S metres, of a body from a fixed point, is given by the equation:

$S = 8t^3 - 33t^2 + 45t$ where t is the time in seconds.

Determine EACH of the following for this body:

(i) its initial velocity; (3)

(ii) the times when it is at rest; (4)

(iii) the time when its acceleration is 30 ms^{-2}. (3)

(b) Determine $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$ for thefunction:

$$v = \frac{1-t^4}{1-t^2}$$ (6)

8. (a) The cross-section of a cargo space in a small bulk carrier can be represented by the area enclosed by the curve $y = \frac{1}{8}x^2$ and the lines $y = 2$ and $y = 8$, as shown by the shaded region in Fig Q8(a).

The units are in metres.

Calculate the area of this cross-section. (10)

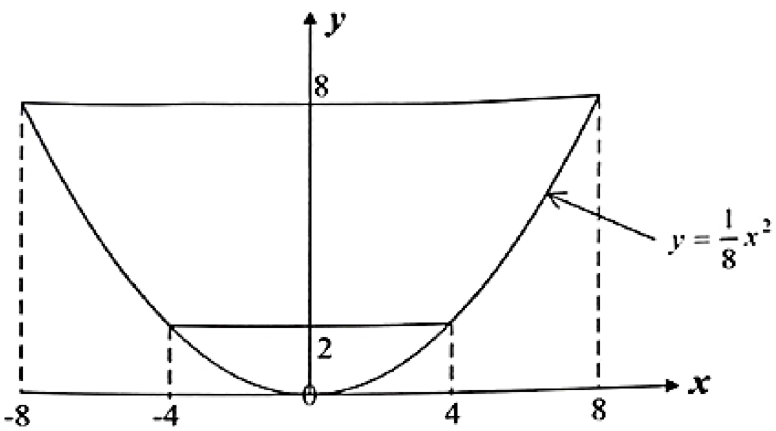

Fig Q8(a)

(b) Given $\frac{dh}{dt} = t^3\left(1 - \frac{4}{t^5}\right) + 3$ and $h = 27$ when $t = 4$, express h as a function of t. (6)